The Large Energy Expansion for B Decays: Soft Collinear Effective Theory

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I this talk I give an introduction to the soft collinear effective theory by considering in detail the decay rate for $B \to X_s\gamma$ near the endpoint.

1 Introduction

Effective field theories provide a simple and elegant method for calculating processes with several relevant energy scales [1, 2, 3, 4, 5, 6]. Part of the utility of effective theories is that they dramatically simplify the summation of logarithms of ratios of mass scales, which would otherwise make perturbation theory poorly behaved. Furthermore the systematic power counting in effective theories, and the approximate symmetries of the effective field theory can greatly reduce the complexity of calculations.

Consider as an example the semi-leptonic decay of a $\overline{B}$-meson to a D-meson. In perturbation theory the one-loop corrections will typically be enhanced by $\log(M/\Lambda_{\rm QCD})$, where M is a heavy quark mass. These logarithms are large enough so that $\alpha_s \log(M/\Lambda_{\rm QCD}) \sim 1$, and the perturbative expansion breaks down. Furthermore the nonperturbative physics in the decay process is parameterized in terms of two unknown form factors.

The power of effective field theories is demonstrated when we consider our example within the context of heavy quark effective theory (HQET) [7]. In HQET heavy particles have been removed from QCD so that logarithms in loop integrals are of the form $\log(\mu/\Lambda_{\rm QCD})$. Furthermore the complete series of leading logarithms $\alpha_s^n \log^n(\mu/\Lambda_{\rm QCD})$ is straightforward to sum via the renormalization group. The HQET Lagrangian has a spin-flavor symmetry which reduces the number of form-factors to a single one: the Isgur-Wise function [8]. In addition HQET tells us the normalization of the Isgur-Wise function at a kinematic point. Remarkably heavy quark spin symmetry leads to additional relations among weak decay form factors. The four form factors which are required to parameterize matrix elements of vector currents and the four axial vector current form factors reduce to the single Isgur-Wise function in the heavy quark limit.

This example clearly illustrates the power of effective field theories. To motivate soft collinear effective theory (SCET) I will consider another example: the decay rate for $\overline{B} \to X_s\gamma$. The dominant contribution to this decay arises from the magnetic penguin operator [9]

$$\hat{O}_7 = \frac{e}{16\pi^2} m_b \, \overline{s} \, \sigma^{\mu\nu} \frac{1}{2}(1+\gamma_5) b \, F_{\mu\nu} \, , \tag{1}$$

where the strange quark mass has been set to zero. The operator product expansion (OPE) for this decay is illustrated in Fig. 1. We write the momenta of the b quark, photon, and light s quark jet as

$$p_b^\mu = m_b v^\mu + k^\mu, \quad q^\mu = \frac{m_b}{2} x \overline{n}^\mu, \quad p_s^\mu = \frac{m_b}{2} n^\mu + l^\mu + k^\mu \tag{2}$$

where, in the rest frame of the B meson,

$$v^\mu = (1, \vec{0}), \quad n^\mu = (1,0,0,-1), \quad \overline{n}^\mu = (1,0,0,1). \tag{3}$$

Here k^μ is a residual momentum of order $\Lambda_{\rm QCD}$, and $l^\mu = \frac{m_b}{2}(1-x)\overline{n}^\mu$, where $x = 2E_\gamma/m_b$. The invariant mass of the light s-quark jet

$$p_s^2 \approx m_b \, n \cdot (l+k) = m_b^2 (1 - x + \hat{k}_+)\,, \tag{4}$$

(where $\hat{k}_+ = k_+/m_b$) is $O(m_b^2)$ except near the endpoint of the photon energy spectrum where $x \to 1$. Inclusive quantities are calculated via the OPE by taking the imaginary part of the graphs on the left hand side of the double arrow in Fig. 1 and expanding in powers of $k^\mu/\sqrt{p_s^2}$. As long as x is not too close to the endpoint, this is an expansion in powers in k^μ/m_b, which matches onto local operators shown graphically on the right hand side of the double arrow in Fig. 1. This leads to an expansion for the photon energy spectrum as a function of x in powers

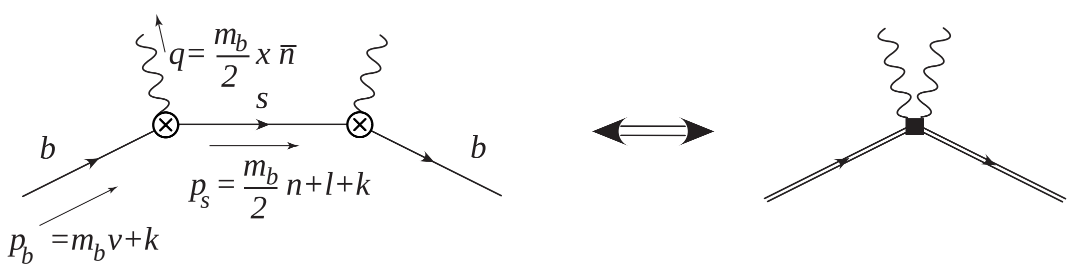

Figure 1: The OPE for $B \to X_s \gamma$.

of α_s and $1/m_b$ [10, 11]:

$$\begin{aligned}
\frac{d\Gamma}{dx} &= \Gamma_0(\mu) \left[\frac{m_b(\mu)}{m_b}\right]^2 \left\{ \left[1 - \frac{\alpha_s C_F}{4\pi}\left(2\log\frac{\mu^2}{m_b^2} + 5 + \frac{4}{3}\pi^2\right)\right]\delta(1-x) \right.\\
&\quad + \frac{\alpha_s C_F}{4\pi}\left[7 + x - 2x^2 - 2(1+x)\log(1-x) - \left(4\frac{\log(1-x)}{1-x} + \frac{7}{1-x}\right)_+\right]\\
&\quad \left. + \frac{1}{2m_b^2}\left[(\lambda_1 - 9\lambda_2)\delta(1-x) - (\lambda_1 + 3\lambda_2)\delta'(1-x) - \frac{\lambda_1}{3}\delta''(1-x)\right]\right\}\\
&\quad + O(\alpha_s^2, 1/m_b^3),
\end{aligned} \tag{5}$$

where

$$\Gamma_0(\mu) = \frac{G_F^2 |V_{tb}V_{ts}^*|^2 \alpha |C_7(\mu)|^2}{32\pi^4} m_b^5\,, \tag{6}$$

and the subscript "+" denotes the usual plus distribution. The parameters λ_1 and λ_2 are matrix elements of local dimension five operators.

Near the endpoint of the photon spectrum, $x \to 1$, both the perturbative and nonperturbative corrections are singular and the OPE breaks down. The severity of the breakdown is most easily seen by integrating the spectrum over a region $1 - \Delta < x < 1$. When $\Delta \leq \Lambda_{\rm QCD}/m_b$ the most singular terms in the $1/m_b$ expansion sum up into a nonperturbative shape function of characteristic width $\Lambda_{\rm QCD}/m_b$[12, 13]. The perturbative series is of the form

$$\frac{1}{\Gamma_0}\int_{1-\Delta}^{1}\frac{d\Gamma}{dx} = 1 + \frac{\alpha_s C_F}{4\pi}\left(-2\log^2\Delta - 7\log\Delta + \ldots\right) + O\left(\alpha_s^2\right), \tag{7}$$

where the ellipses denote terms that are finite as $\Delta \to 0$. These Sudakov logarithms are large for $\Delta \ll 1$, and can spoil the convergence of perturbation theory. The full series has been shown to exponentiate [14, 15], which sums the leading and next-to-leading logarithms.

In general, "phase space" logarithms are to be expected whenever a decay depends on several distinct scales. For example, in $b \to X_c e\overline{\nu}_e$ decay the rate calculated with the OPE performed at $\mu = m_b$ contains logarithms of m_c/m_b, which become large in the $m_b \gg m_c$ limit. In [16] an effective theory was used to run from m_b to m_c, summing phase space logarithms of the ratio m_c/m_b. Similarly, in $b \to X_s\gamma$ near the endpoint of the photon energy spectrum the invariant mass of the light quark jet scales as $m_b\sqrt{1-x}$, and is widely separated from the scale $\mu = m_b$ where the OPE is performed. In order to sum logarithms of Δ (or the more complicated plus distributions in the differential spectrum, Eq. (5)) we would expect to have to switch to a new effective theory at $\mu = m_b$, use the renormalization group to run down to a scale of order $m_b\sqrt{1-x}$, at which point the OPE is performed. In fact, we will see that the situation is more complicated than this.

We are then left with the question of the appropriate theory below the scale m_b. To see where we might begin let us return to the expressions for the s-quark momentum and the invariant mass given in Eq. (2) and Eq. (4) respectively. For $1 - x \sim \mathcal{O}(\Lambda_{\rm QCD}/m_b)$ we find $l^\mu \sim k^\mu \sim \Lambda_{\rm QCD}$, and the invariant mass of the s-quark is $\mathcal{O}(m_b\Lambda_{\rm QCD})$. On the other hand the momentum of the s-quark has a large component of order m_b in the light-cone direction n^μ with residual momentum of order $\Lambda_{\rm QCD}$. Thus in this kinematic region the s-quark is light-like. Given that the form of the light-like momentum, $p_s^\mu = (m_b/2)n^\mu + \tilde{k}^\mu$ with $\tilde{k}^\mu \sim \Lambda_{\rm QCD}$, is the same as the form of the heavy quark momentum, $p_b^\mu = m_b v^\mu + k^\mu$, with the time like vector v^μ replaced with the light-like vector n^μ it is very tempting to introduce an effective theory of light-like Wilson lines, much as HQET is an effective theory of time-like Wilson lines[17]. Such an effective theory, christened the large energy effective theory (LEET), was proposed many years ago by Dugan and Grinstein[18]. However, a simple attempt at matching shows that LEET does not reproduce the infrared physics of QCD [19]. The problem is that LEET only describes the coupling of light-like particles to soft gluons, but does not describe the splitting of an energetic particle into two almost collinear particles.

2 Soft Collinear Effective Theory

In the rest frame of the heavy hadron the light particles in the decay move close to the light cone direction n^μ and their dynamics is best described in terms of light cone coordinates

$p = (p^+, p^-, p_\perp)$, where $p^+ = n \cdot p$, $p^- = \overline{n} \cdot p$. For large energies the different light cone components are widely separated, with $p^- \sim Q$ being large, while $p_\perp$ and p^+ are small. Taking the small parameter to be $\lambda \sim p_\perp / p^-$ we have

$$p^\mu = \overline{n} \cdot p \, \frac{n^\mu}{2} + p_\perp^\mu + n \cdot p \, \frac{\overline{n}^\mu}{2} = \mathcal{O}(\lambda^0) + \mathcal{O}(\lambda^1) + \mathcal{O}(\lambda^2)\,, \tag{8}$$

where we have used $p^+ p^- \sim p_\perp^2 \sim Q^2 \lambda^2$ for fluctuations near the mass shell. Thus the light-cone momentum components of collinear particles scale like $k_c = Q(\lambda^2, 1, \lambda)$. The collinear quark can emit either a gluon collinear to the large momentum direction or a gluon with momentum scaling $k_{us} = Q(\lambda^2, \lambda^2, \lambda^2)$ (referred to as an ultra-soft or usoft gluon). For scales above the typical off-shellness of the collinear degrees of freedom, $k_c^2 \sim (Q\lambda)^2$, both gluon modes are required to correctly reproduce all the infrared physics of the full theory. This was described in [19], where it was shown that at a scale $\mu \sim Q$, QCD can be matched onto an effective theory that contains heavy quarks and light collinear quarks, as well as usoft and collinear gluons.

The SCET Lagrangian can be obtained at tree level by expanding the full theory Lagrangian in powers of λ [20]. We start from the QCD Lagrangian for massless quarks and gluons

$$\mathcal{L}_{\rm QCD} \;=\; \overline{\psi}\, i \not{D}\, \psi\,, \tag{9}$$

where the covariant derivative is $D_\mu = \partial_\mu - igT^a A_\mu^a$. We begin by removing the large momenta from the effective theory fields, similar to the construction of HQET. In HQET there are two relevant momentum scales, the mass of the heavy quark m and $\Lambda_{\rm QCD}$. The scale m is separated from $\Lambda_{\rm QCD}$ by writing $p = mv + k$, where $v^2 = 1$ and the residual momentum $k \ll m$. The variable v becomes a label on the effective theory fields. Our case is slightly more complicated because there are three scales to consider. We split the momenta p by taking

$$p = \tilde{p} + k\,, \qquad \text{where} \quad \tilde{p} \equiv \frac{1}{2}(\overline{n} \cdot p) n + p_\perp\,. \tag{10}$$

The "large" parts of the quark momentum $\overline{n} \cdot p \sim 1$ and $p_\perp \sim \lambda$, denoted by $\tilde{p}$, become a label on the effective theory field, while the residual momentum $k^\mu \sim \lambda^2$ is dynamical.

The large momenta $\tilde{p}$ are removed by defining a new field $\psi_{n,p}$ through

$$\psi(x) = \sum_{\tilde{p}} e^{-i\tilde{p}\cdot x} \psi_{n,p}\,. \tag{11}$$

A label p is given to the $\psi_{n,p}$ field, with the understanding that only the components $\overline{n} \cdot p$ and $p_\perp$ are labels. Derivatives ∂^μ on the field $\psi_{n,p}(x)$ give order λ^2 contributions. For a particle moving along the n^μ direction, the four component field $\psi_{n,p}$ has two large components $\xi_{n,p}$ and two small components $\xi_{\overline{n},p}$. These components can be obtained from the field $\psi_{n,p}$ using projection operators

$$\xi_{n,p} = \frac{\not{n}\not{\overline{n}}}{4}\, \psi_{n,p}\,, \qquad \xi_{\overline{n},p} = \frac{\not{\overline{n}}\not{n}}{4}\, \psi_{n,p}\,, \tag{12}$$

and satisfy the relations

$$\begin{aligned} &\frac{\not{n}\not{\overline{n}}}{4}\, \xi_{n,p} = \xi_{n,p}\,, \quad \not{n}\, \xi_{n,p} = 0\,, \\ &\frac{\not{\overline{n}}\not{n}}{4}\, \xi_{\overline{n},p} = \xi_{\overline{n},p}\,, \quad \not{\overline{n}}\, \xi_{\overline{n},p} = 0\,. \end{aligned} \tag{13}$$

In terms of these fields the quark part of the QCD Lagrangian given in Eq. (9) becomes

$$\mathcal{L} = \sum_{\tilde{p},\tilde{p}'} \Big[\overline{\xi}_{n,p'} \frac{\not{\overline{n}}}{2} \Big(in\cdot D \Big) \xi_{n,p} + \overline{\xi}_{\overline{n},p'} \frac{\not{n}}{2} \Big(\overline{n}\cdot p + i\overline{n}\cdot D \Big) \xi_{\overline{n},p} + \overline{\xi}_{n,p'} \Big(\not{p}_\perp + i\not{D}_\perp \Big) \xi_{\overline{n},p} + \overline{\xi}_{\overline{n},p'} \Big(\not{p}_\perp + i\not{D}_\perp \Big) \xi_{n,p} \Big] . \tag{14}$$

Since the derivatives on the fermionic fields yield momenta of order $k \sim \lambda^2$ they are suppressed relative to the labels $\overline{n}\cdot p$ and $p_\perp$. Without the $\overline{n}\cdot D$ and $D_\perp$ derivatives, $\xi_{\overline{n},p}$ is not a dynamical field. Thus, we can eliminate $\xi_{\overline{n},p}$ at tree level by using the equation of motion

$$(\overline{n}\cdot p + \overline{n}\cdot iD)\xi_{\overline{n},p} = (\not{p}_\perp + i\not{D}_\perp)\frac{\not{\overline{n}}}{2}\xi_{n,p} . \tag{15}$$

Eqs. (14) and (15) result in a Lagrangian involving only the two components $\xi_{n,p}$:

$$\mathcal{L} = \sum_{\tilde{p},\tilde{p}'} e^{-i(\tilde{p}-\tilde{p}')\cdot x} \overline{\xi}_{n,p'} \left[n\cdot iD + (\not{p}_\perp + i\not{D}_\perp) \frac{1}{\overline{n}\cdot p + \overline{n}\cdot iD} (\not{p}_\perp + i\not{D}_\perp) \right] \frac{\not{\overline{n}}}{2} \xi_{n,p} . \tag{16}$$

Here the summation extends over all distinct copies of the fields labelled by $\tilde{p}, \tilde{p}'$. The gluon field in D^μ includes collinear and usoft parts, $A^\mu \to A^\mu_c + A^\mu_{us}$. The two types of gluons are distinguished by the length scales over which they fluctuate. Fluctuations of the collinear gluon fields A^μ_c are characterized by the scale $q^2 \sim \lambda^2$, while fluctuations of the usoft gluon field A^μ_{us} are characterized by $k^2 \sim \lambda^4$. Since the collinear gluon field has large momentum components $\tilde{q} \equiv (\overline{n}\cdot q, q_\perp)$, derivatives acting on these fields can still give order $\lambda^{0,1}$ contributions. To make this explicit we label the collinear gluon field by its large momentum components $\tilde{q}$, and extract the phase factor containing $\tilde{q}$ by redefining the field: $A_c(x) \to e^{-i\tilde{q}\cdot x} A_{n,q}(x)$. Inserting this into Eq. (16) one finds

$$\begin{aligned}\mathcal{L} = \sum_{\tilde{p},\tilde{p}',\tilde{q}} e^{-i(\tilde{p}-\tilde{p}')\cdot x} \overline{\xi}_{n,p'} \Big[& n\cdot iD + g e^{-i\tilde{q}\cdot x} n\cdot A_{n,q} + \Big(\not{p}_\perp + i\not{D}_\perp + g e^{-i\tilde{q}\cdot x} \not{A}^\perp_{n,q} \Big) \\ & \times \frac{1}{\overline{n}\cdot p + \overline{n}\cdot iD + g e^{-i\tilde{q}\cdot x} \overline{n}\cdot A_{n,q}} \Big(\not{p}_\perp + i\not{D}_\perp + g e^{-i\tilde{q}\cdot x} \not{A}^\perp_{n,q} \Big) \Big] \frac{\not{\overline{n}}}{2} \xi_{n,p} .\end{aligned} \tag{17}$$

The covariant derivative is defined to only involve usoft gluons. We immediately notice a problem: derivatives in the the last term in brackets above can act on the phase factors associated with the collinear gluon fields. Thus these derivatives can result in terms of order λ^0, λ^1, while we want all derivatives to scale as λ^2.

We need to split up the derivative into a piece that acts on the large components of the collinear momentum, and a residual piece that is $\mathcal{O}(\lambda^2)$. Towards this end we introduce a projection operator $\mathcal{P}$ which only acts on the large label of the collinear fields [21]. For any function f

$$\begin{aligned} & f(\overline{\mathcal{P}}) \phi^\dagger_{q_1} \cdots \phi^\dagger_{q_m} \phi_{p_1} \cdots \phi_{p_n} \\ & \quad f(\overline{n}\cdot p_1 + \ldots + \overline{n}\cdot p_n - \overline{n}\cdot q_1 - \ldots \overline{n} - \cdot q_m) \phi^\dagger_{q_1} \cdots \phi^\dagger_{q_m} \phi_{p_1} \cdots \phi_{p_n} ,\end{aligned} \tag{18}$$

	heavy quark	collinear quark	usoft gluon	collinear gluons		
Field	h_v	$\xi_{n,p}$	A^μ_{us}	$\overline{n}\cdot A_{n,q}$	$n\cdot A_{n,q}$	$A^\perp_{n,q}$
Scaling	λ^3	λ	λ^2	λ^0	λ^2	λ

Table 1: Power counting for SCET fields.

where $\overline{\mathcal{P}} \equiv \overline{n}\cdot\mathcal{P}$. Then we have

$$i\partial^\mu e^{-ip\cdot x}\phi_{n,p}(x) = e^{-ip\cdot x}(\mathcal{P}^\mu + i\partial^\mu)\phi_{n,p}(x)\,, \tag{19}$$

and the phases involving large momentum components can be removed from the Lagrangian in Eq. 17 as long as we adopt a convention that there is an implicit sum over labels, and that total label momentum is conserved.

Finally, we expand Eq. (17) in powers of λ. To simplify the power counting we follow the procedure of moving all the dependence on λ into the interaction terms of the action to make the kinetic terms of order λ^0 [22, 23, 24]. This is done by assigning a λ scaling to the effective theory fields as given in Table 1. The power counting in Table 1 gives an order one kinetic term for collinear gluons in an arbitrary gauge. In generalized covariant gauge

$$\int d^4x\, e^{ik\cdot x}\,\langle 0|\, T\, A^\mu_c(x)A^\nu_c(0)\,|0\rangle = \frac{-i}{k^2}\Big(g^{\mu\nu} - \alpha\frac{k^\mu k^\nu}{k^2}\Big) \tag{20}$$

and the scaling of the components on the right and left hand side of this equation agree. Note that x must be rescaled as well: $(x^+, x^-, x^\perp) \to (x^+/\lambda^2, x^-, x^\perp/\lambda)$. With this power counting all interactions scale as λ^n with $n \geq 0$, and the leading SCET Lagrangian for the collinear quark sector is

$$\mathcal{L}^{(0)}_{\xi\xi} = \overline{\xi}_{n,p'}\Big\{in\cdot D + i\not{D}^\perp_c\frac{1}{i\overline{n}\cdot D_c}i\not{D}^\perp_c\Big\}\frac{\not{\overline{n}}}{2}\xi_{n,p}\,, \tag{21}$$

where $in\cdot D = in\cdot\partial + gn\cdot A_{n,q} + gn\cdot A_{us}$, $i\overline{n}\cdot D_c = \overline{\mathcal{P}} + g\overline{n}\cdot A_{n,q}$, and $iD^{\perp\mu}_c = \mathcal{P}^{\perp\mu} + gA^{\perp\mu}_{n,q}$.

As I mentioned at the beginning of the talk an important aspect of effective field theories is the approximate symmetries that are manifest in the leading order Lagrangian. The SCET Lagrangian presented above has a global helicity spin symmetry, which for example can lead to a reduction in the number of form factors needed to parameterize heavy-to-light decays. In addition SCET has a powerful set of gauge symmetries [25]. Specifically the collinear and usoft fields each have their own gauge transformation that leave the Lagrangian invariant. Collinear gauge transformations are the subset of QCD gauge transformations where $\partial^\mu U(x) \sim Q(\lambda^2, 1, \lambda)$, and usoft gauge transformations are those where $\partial^\mu V(x) \sim Q\lambda^2$. The invariance under each of these transformations is a manifestation of scales of order Q or greater having been removed from the theory, since any gauge transformation that would change a usoft gluon into a collinear gluon would imply a boost of order Q. The gauge transformations for the SCET fields are shown in Table 2. The usoft field acts like a classical background field in which the collinear particle propagates, and the collinear fields transform similarly to a global color-rotation under a usoft gauge transformation. In a moment we will see why the gauge invariance

Fields	Collinear U	Usoft V
$\xi_{n,p}$	$U\ \xi_{n,p}$	$V\,\xi_{n,p}$
$A^\mu_{n,q}$	$U\ A^\mu_{n,q}\,U^\dagger + \frac{1}{g}\,U\Big[i\mathcal{D}^\mu\ U^\dagger\Big]$	$V\ A^\mu_{n,q}\,V^\dagger$
q_{us}	q_{us}	$V\ q_{us}$
A^μ_{us}	A^μ_{us}	$V\Big(A^\mu_{us} + \frac{i}{g}\partial^\mu\Big)V^\dagger$

Table 2: Gauge transformations for the collinear and usoft fields. Here $i\mathcal{D}^\mu \equiv (n^\mu/2)\overline{\mathcal{P}} + \mathcal{P}^\mu_\perp + (\overline{n}^\mu/2)in\cdot D$.

structure is so powerful. But first I want to comment on something called reparameterization invariance.

Reparametrization invariance (RPI) [26, 27] in SCET is a manifestation of the Lorentz symmetry that was broken by introducing the vectors n and $\overline{n}$. Lorentz symmetry tells us that any choice of light-like vectors n and $\overline{n}$ is equally good as long as $n^2 = 0$, $\overline{n}^2 = 0$, and $n\cdot\overline{n} = 2$. This implies that SCET operators must be invariant under the most general set of transformations that satisfy the conditions just enumerated. These transformations fall into three catagories: I) $n \to n + \Delta_\perp$, $\overline{n} \to \overline{n}$, II) $n \to n$, $\overline{n} \to \overline{n} + \epsilon_\perp$, and III) $n \to e^\alpha n$, $\overline{n} \to e^{-\alpha}\overline{n}$, where $\Delta_\perp \sim \lambda$ and $\epsilon_\perp, \alpha \sim \lambda^0$. Requiring invariance of operators under these transformations results in powerful constraints on the forms of operators. In fact RPI is an essential tool for deducing subleading corrections in SCET [28].

3 Heavy-to-Light current

Next I discuss matching the SCET heavy to light currents. To perform the matching, first consider the simpler case of an Abelian gauge group. In this case calculating the full theory graph with m gluons in Fig. 2, expanding in powers of λ, and putting the result over a common denominator gives

$$c_m(\mu = m_b) = \frac{1}{m!}\prod_{i=1}^{m}\frac{1}{\overline{n}\cdot q_i}\,. \tag{22}$$

The factor of $1/m!$ is from the presence of m identical A_c fields at the same point. Thus, we have the tree level result

$$J^{\rm eft}_{\rm had}\Big|_{\mu=m_b} \;=\; \overline{\xi}_{n,p}\,\exp\left(\frac{g\,\overline{n}\cdot A_{n,q}}{\overline{n}\cdot q}\right)\Gamma\,h_v\,. \tag{23}$$

We can rewrite the exponential in the above expression using the projection operator $\mathcal{P}$:

$$\exp\left(\frac{g\,\overline{n}\cdot A_{n,q}}{\overline{n}\cdot q}\right) \;=\; \sum_{\rm perms}\exp\left(\frac{g\,\overline{n}\cdot A_{n,q}}{\overline{\mathcal{P}}}\right) \equiv W^\dagger\,. \tag{24}$$

Where we have given the above object a name since it will occur again and again. Why is W so important? Consider a collinear gauge transformation on the current in Eq. (23). The field

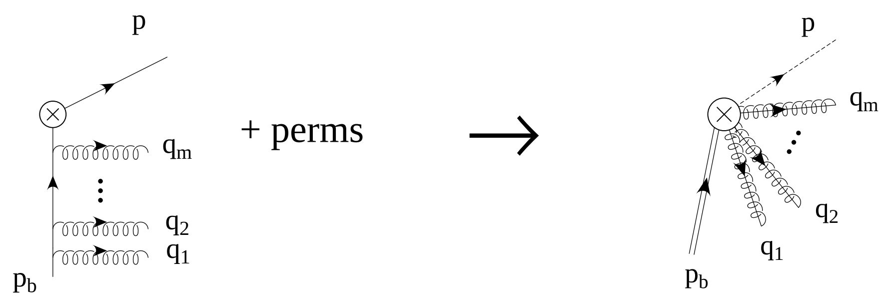

Figure 2: Matching for the order λ^0 Feynman rule for the heavy to light current with n collinear gluons. All permutations of crossed gluon lines are included on the left.

h_v is invariant since collinear gluons do not couple to heavy quarks, on the other hand, the collinear quark field transforms as $\xi_{n,p} \to e^{i\alpha(x)}\xi_{n,p}$. Thus, the operator $\overline{\xi}_{n,p}\Gamma h_v$ is not gauge invariant. However, we find that

$$\exp\left(\frac{g\,\overline{n}\cdot A_{n,q}}{\overline{n}\cdot q}\right) \to \exp\left(\frac{g\,\overline{n}\cdot A_{n,q}}{\overline{n}\cdot q}\right)\exp\Big[i\alpha(x)\Big]\,, \tag{25}$$

and the last exponential exactly cancels the transformation of $\overline{\xi}_{n,p}$. By gauge invariance the current therefore has to be of the form in Eq. (23) for an arbitrary scale μ. It is convenient to define a field that transforms as a singlet under a collinear gauge transformation

$$\chi_n = W\xi_{n,p}\,. \tag{26}$$

We will refer to χ_n as the jet field since it involves a collinear quark field plus an arbitrary number of collinear gluons moving in the n direction.

Hard fluctuations in the full theory do not occur in the effective theory since they have been integrated out. However, the physics of the hard fluctuations appears in the Wilson coefficients of the effective theory as a result of matching. The SCET Wilson coefficients can therefore be nontrivial functions of the large collinear momentum, $C(\overline{n}\cdot p_i)$. Fortunately collinear gauge invariance restricts these coefficients so that they only depend on the linear combination picked out by $\overline{\mathcal{P}}$. Thus the general Wilson coefficient in SCET will be a function $C(\overline{\mathcal{P}},\overline{\mathcal{P}}^\dagger)$ which must be inserted between gauge invariant products of collinear fields and W. Thus in terms of the χ_n field the leading order effective theory current for $Q\lambda < \mu < m_b$ has the form

$$J^{\rm eft}_{\rm had} \;=\; \overline{\chi}_n\, C_i(\mu,\overline{\mathcal{P}}^\dagger)\,\Gamma\, h_v\,, \tag{27}$$

where $\overline{\mathcal{P}}^\dagger$ acts to the left. For a non-abelian gauge group a similar gauge invariance argument applies, however the matching in Fig. 2 is more complicated. In momentum space we find

$$\begin{aligned}\chi_n \;&=\; \sum_k\sum_{\rm perms}\frac{(-g)^k}{k!}\left(\frac{\overline{n}\cdot A_{\overline{n},q_1}\cdots\overline{n}\cdot A_{\overline{n},q_k}}{[\overline{n}\cdot q_1][\overline{n}\cdot(q_1+q_2)]\cdots[\overline{n}\cdot\sum_{i=1}^k q_i]}\right)\xi_{n,p}\\ &=\;\sum_{\rm perms}\exp\left(\frac{-g\,\overline{n}\cdot A_{n,q}}{\overline{\mathcal{P}}}\right)\xi_{n,p} = W\xi_{n,p}\,.\end{aligned} \tag{28}$$

4 Factorization

A remarkable consequence of the gauge symmetries of SCET is the factorization of usoft and collinear effects. Towards this end we introduce the usoft Wilson line

$$Y(x) = \mathrm{Pexp}\Big(ig\int_{-\infty}^{x} ds\, n\cdot A_{us}(ns)\Big)\,, \tag{29}$$

and redefine the collinear fields as follows:

$$\xi_{n,p} = Y\xi_{n,p}^{(0)} \qquad A^{\mu}_{n,q} = Y A^{(0)\mu}_{n,q} Y^{\dagger}\,. \tag{30}$$

Under these field redefinitions the usoft gluons decouple from the collinear fields. In other words the leading Lagrangian in Eq. (21) becomes completely independent of the usoft fields. At higher orders in the λ expansion this decoupling does not occur. Furthermore as we will see decoupling of usoft gluons in the leading Lagrangian does not necessarily mean that usoft gluons decouple in subleading operators or currents.

Let us now return to our example and study the consequences of the above field redefinition. The inclusive photon energy spectrum can be written using the optical theorem as

$$\frac{1}{\Gamma_0(m_b)}\frac{d\Gamma}{dE_\gamma} = \frac{4E_\gamma}{m_b^3}\Big(-\frac{1}{\pi}\Big)\,\mathrm{Im}\,T(E_\gamma)\,, \tag{31}$$

where the forward scattering amplitude $T(E_\gamma)$ is

$$T(E_\gamma) = \frac{i}{m_B}\int d^4x\, e^{-iq\cdot x}\,\langle \overline{B}|T J^{\dagger}_{\mu}(x) J^{\mu}(0)|\overline{B}\rangle\,, \tag{32}$$

with relativistic normalization for the $|\overline{B}\rangle$ states. The current is $J_\mu = \overline{s}\, i\sigma_{\mu\nu}q^\nu P_R\, b$, and $\Gamma_0(m_b)$ is given in Eq. (6)

In the endpoint region we match the current in the time ordered product in Eq. (32) onto SCET fields. At leading order in λ

$$J_\mu \;=\; -E_\gamma\, e^{i(\overline{\mathcal{P}}\frac{n}{2}+\mathcal{P}_\perp - m_b v)\cdot x}\Big\{[2C_9(\overline{\mathcal{P}},\mu)+C_{12}(\overline{\mathcal{P}},\mu)]J^{\rm eff}_\mu - C_{10}(\overline{\mathcal{P}},\mu)\tilde{J}^{\rm eff}_\mu\Big\}\,, \tag{33}$$

where

$$J^{\rm eff}_\mu \;=\; \overline{\xi}_{n,p}\, W\gamma^\perp_\mu P_L\, b_v\,, \qquad \tilde{J}^{\rm eff}_\mu = \overline{n}_\mu \overline{\xi}_{n,p}\, W P_R\, b_v\,. \tag{34}$$

The SCET Wilson coefficients $C_{9,10,12}(\overline{\mathcal{P}},\mu)$ are given at one-loop in Ref. [20]. In Eq. (33) label conservation sets $\overline{\mathcal{P}} = m_b$ and $\mathcal{P}_\perp = 0$. The current $\tilde{J}^{\rm eff}_\mu$ does not contribute for real transversely polarized photons so I drop it. Inserting Eq. (33) into Eq. (32) gives

$$\frac{4E_\gamma}{m_b^3}\,T(E_\gamma) \equiv H(m_b,\mu)\,T^{\rm eff}(E_\gamma,\mu)\,, \tag{35}$$

where $T^{\rm eff}$ is the forward scattering amplitude in the effective theory

$$T^{\rm eff} \;=\; i\int d^4x\, e^{i(m_b\frac{\overline{n}}{2}-q)\cdot x}\,\Big\langle \overline{B}_v\Big|\,\mathrm{T}\,J^{\rm eff\dagger}_{\mu}(x)\,J^{\rm eff\mu}(0)\,\Big|\overline{B}_v\Big\rangle\,, \tag{36}$$

with HQET normalization for the states. The hard amplitude is

$$H(m_b,\mu) = \frac{16E_\gamma^3}{m_b^3}\left|C_9(m_b,\mu) + \frac{1}{2}C_{12}(m_b,\mu)\right|^2 . \tag{37}$$

Next the usoft gluons are decoupled from the collinear fields by the field redefinitions in Eq. (30) along with

$$W \to Y W^{(0)} Y^\dagger\,, \tag{38}$$

which is a consequence of Eq. (30). This gives

$$J_\mu^{\rm eff} = \overline{\xi}^{(0)}_{n,p}\, W^{(0)}\gamma_\mu^\perp P_L\, Y^\dagger\, b_v\,, \tag{39}$$

Thus, the time-ordered product of the effective theory currents is

$$\begin{aligned} T^{\rm eff} \;=\; & i\int d^4x\, e^{i(m_b\frac{\overline{n}}{2}-q)\cdot x}\left\langle \overline{B}_v\right| \mathrm{T}\,[\overline{b}_v Y](x)\,[Y^\dagger b_v](0)\left|\overline{B}_v\right\rangle \\ & \times\left\langle 0\right| \mathrm{T}\,[W^{(0)\dagger}\xi^{(0)}_{n,p}](x)\,[\overline{\xi}^{(0)}_{n,p}W^{(0)}](0)\left|0\right\rangle . \end{aligned} \tag{40}$$

In the effective theory the Hilbert space of states is the direct product of the usoft and collinear Hilbert states. As a consequence the $\overline{B}$ meson state contains no collinear particles, and the collinear physics can be separated from the usoft physics. Next introduce the Fourier transform

$$\left\langle 0\right| \mathrm{T}\,[W^{(0)\dagger}\xi^{(0)}_{n,p}](x)\,[\overline{\xi}^{(0)}_{n,p}W^{(0)}](0)\left|0\right\rangle \equiv i\int\frac{d^4k}{(2\pi)^4}\, e^{-ik\cdot x}\, J(P,k)\,\frac{\not{n}}{2}\,, \tag{41}$$

where P is sum of the label momenta carried by the collinear fields. By momentum conservation $P = m_b$. Because the collinear Lagrangian contains only the $n\cdot\partial$ derivative $J(p,k)$ only depends on the component $k^+ = n\cdot k$ of the residual momentum k. This allows us to perform the $k_-, k_\perp$ integrations in Eq. (41) which puts x on the light cone

$$\begin{aligned} T^{\rm eff} &= \frac{1}{2}\int d^4x\, e^{i(m_b\frac{\overline{n}}{2}-q)\cdot x}\,\delta(x^+)\delta(\vec{x}_\perp)\int\frac{dk^+}{2\pi}\, e^{-\frac{i}{2}k^+x^-}\left\langle \overline{B}_v\right| \mathrm{T}\,[\overline{b}_v Y](x)\,[Y^\dagger b_v](0)\left|\overline{B}_v\right\rangle J(P,k^+) \\ &= \frac{1}{2}\int dk^+\, J(P,k^+)\int\frac{dx^-}{4\pi}\, e^{-\frac{i}{2}(2E_\gamma - m_b + k^+)x^-}\left\langle \overline{B}_v\right| \mathrm{T}\,[\overline{b}_v Y](\tfrac{n}{2}\,x^-)\,[Y^\dagger b_v](0)\left|\overline{B}_v\right\rangle . \end{aligned} \tag{42}$$

The typical offshellness of the collinear particles is $p^2 \sim m_b\Lambda_{\rm QCD}$ so the function $J(P,k^+)$ can be calculated perturbatively. At lowest order in $\alpha_s(\sqrt{m_b\Lambda_{\rm QCD}})$, $J(P,k^+)$ is determined by the collinear quark propagator carrying momentum $(P+k)$

$$J(P,k^+) = \frac{\overline{n}\cdot P}{(P+k)^2 + i\epsilon} = \frac{1}{n\cdot k + P_\perp^2/(\overline{n}\cdot P) + i\epsilon}\,. \tag{43}$$

The remaining matrix element in Eq. (42) is purely usoft

$$S(l^+) \;\equiv\; \frac{1}{2}\int\frac{dx^-}{4\pi}\, e^{-\frac{i}{2}\,l^+x^-}\left\langle \overline{B}_v\right| T\,[\overline{b}_v Y](\tfrac{n}{2}\,x^-)\,[Y^\dagger b_v](0)\left|\overline{B}_v\right\rangle . \tag{44}$$

Inserting this into Eq. (42) and taking the imaginary part gives

$$\frac{1}{\Gamma_0}\frac{d\Gamma}{dE_\gamma} = H(m_b,\mu)\int_{2E_\gamma-m_b}^{\overline{\Lambda}} dk^+\, S(k^+,\mu)\, J(k^+ + m_b - 2E_\gamma,\mu)\,. \tag{45}$$

where

$$J(k^+) \equiv -\frac{1}{\pi}\,\mathrm{Im}\, J(P,k^+)\,. \tag{46}$$

This result is valid to all orders in α_s and leading order in $\Lambda_{\rm QCD}/Q$ where $Q = E_\gamma$ or m_b. The individual terms in Eq. (45) depend on the scale μ in such a way that the decay rate is μ independent. In the next section I discuss the μ dependence of S and J in detail.

5 Renormalization Group: Summing Logarithms

Eq. (5) gives the tree level and α_s corrections to the $B \to X_s + \gamma$ decay rate. The leading order contribution is proportional to $\delta(1-x)$, and the next-to-leading order corrections have contributions of the form $\alpha_s \ln(1-x)/(1-x)$, where $x = 2E_\gamma/m_b$. Clearly when $x \sim 1-\alpha_s$ these corrections are large and must be resummed. This can be accomplished in a straightforward manner by using the renormalization group equations of SCET. The resummation is most easily carried out by taking moments with respect to x, then the large corrections as $x \to 1$ become large logs of N in the expression for the Nth moment.

Taking moments of the factored decay rate in Eq. (45) gives

$$\begin{aligned}\int_0^1 dx\, x^N \frac{1}{\Gamma_0}\frac{d\Gamma}{dx} &= \tilde{H}(m_b,\mu)\int_0^1 dx\, x^N \int_x^1 d\xi\, S(\xi,\mu)\, J(m_b(\xi-x),\mu) \\ &= \tilde{H}(m_b,\mu)\int_0^1 d\xi\, \xi^{N+1}\int_0^1 du\, u^N\, S(\xi,\mu)\, J(m_b\xi(1-u),\mu)\,,\end{aligned} \tag{47}$$

where $\xi = k^+/m_b + 1$. In the last line we made the substitution $x = u\,\xi$. Since the large logs come from the region $\xi, x \sim 1$, the factor of ξ in the argument of the jet function can be set equal to 1. Then the moments factor:

$$\Gamma_N = H(m_b,\mu)\, S_N(\mu)\, J_N(\mu)\,, \tag{48}$$

where

$$\begin{aligned}\Gamma_N &= \int_0^1 dx\, x^N\, \frac{1}{\Gamma_0}\frac{d\Gamma}{dx}\,, \\ S_N &= \int_0^1 d\xi\, \xi^N\, S(\xi,\mu)\,, \\ J_N(\mu) &= \int_0^1 du\, u^N\, J(m_b(1-u),\mu)\,.\end{aligned} \tag{49}$$

We are only interested in the large N moments, so have used $S_{N+1} = S_N + \mathcal{O}(1/N)$.

Figure 3: *Graphs needed to calculate the $\mathcal{O}(\alpha_s)$ counterterm to J_N.*

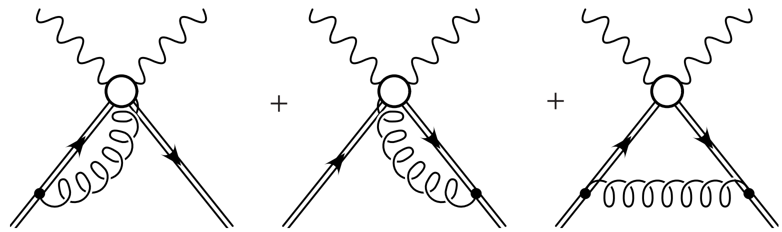

Figure 4: *Graphs needed to calculate the $\mathcal{O}(\alpha_s)$ counterterm to S_N.*

To resum logarithms we use the renormalization group equations for S_N and J_N The one loop anomalous dimension for the jet function renormalization group equations is calculated from the diagrams in Fig. (3). The result is

$$\mu \frac{d}{d\mu} J_N(\mu) = \left[\frac{2C_F\alpha_s}{\pi} \log\left(\frac{\overline{\mu}^2}{m_b^2} \frac{N}{N_0} \right) + \frac{3\alpha_s}{2\pi} C_F \right] J_N(\mu) \,, \tag{50}$$

where $N_0 = e^{-\gamma}$. The one loop anomalous dimension of S_N is determined from the diagrams in Fig (4), and the renormalization group equation immediately follows:

$$\mu \frac{d}{d\mu} S_N(\mu) = \left[-\frac{2C_F\alpha_s}{\pi} \log\left(\frac{\overline{\mu}}{m_b} \frac{N}{N_0} \right) + \frac{\alpha_s C_F}{\pi} \right] S_N(\mu) \,. \tag{51}$$

Defining $y_0 = (N_0/N)$ logarithms in the hard, jet and usoft functions are minimized at the scales $m_b, m_b\sqrt{y_0}$ and $m_b y_0$ respectively. Large logarithms of N are summed by evolving the jet and usoft functions to these scales. The evolution can also be done in one step by defining separate renormalization scales for collinear and usoft loops [29]. Loops whose momenta scale like $(1, \lambda^2, \lambda)$ come with a factor of μ_c^{4-D} and loops whose momenta scale like $(\lambda^2, \lambda^2, \lambda^2)$ come with a factor μ_u^{4-D}. This idea is similar to the velocity renormalization group in NRQCD [30]. The renormalization group equations for J_N and S_N take the form

$$\begin{aligned} \mu_c \frac{d}{d\mu_c} J_N &= \gamma_J^N(\mu_c) J_N \,, \\ \mu_u \frac{d}{d\mu_u} S_N &= \gamma_S^N(\mu_u) S_N \,. \end{aligned} \tag{52}$$

Factorization of usoft and collinear degrees of freedom guarantees that γ_J is a function of μ_c only and that γ_S is a function of μ_u only. The scales are however correlated, so that $\mu_c = m_b\sqrt{y}$ and $\mu_u = m_b y$. Evolving the variable y from 1 to y_0 simultaneously resums large logs in both J_N and S_N.

Defining $\tilde{\Gamma}_N = J_N S_N$, the evolution equation for $\tilde{\Gamma}_N$ as function of y is

$$y\frac{d}{dy}\tilde{\Gamma}_N = \left(\frac{1}{2}\gamma_J^N(m_b\sqrt{y}) + \gamma_S^N(m_b y)\right)\tilde{\Gamma}_N\,. \tag{53}$$

This equation is easily integrated to obtain the following expression for the resummed moments:

$$\Gamma_N = H(m_b)S_N(m_b y_0)\, e^{\log(N)g_1(\chi)+g_2(\chi)}, \tag{54}$$

where

$$\begin{aligned} g_1(\chi) &= -\frac{2C_F}{\beta_0\chi}\left[(1-2\chi)\log(1-2\chi) - 2(1-\chi)\log(1-\chi)\right], \qquad (55)\\ g_2(\chi) &= -\frac{8\Gamma_2}{\beta_0^2}\left[-\log(1-2\chi) + 2\log(1-\chi)\right] - \log(1-\chi)\\ &\quad -\frac{2C_F\beta_1}{\beta_0^3}\left[\log(1-2\chi) - 2\log(1-\chi) + \frac{1}{2}\log^2(1-2\chi) - \log^2(1-\chi)\right]\\ &\quad -\frac{2C_F}{\beta_0}\log(1-2\chi) - \frac{4C_F}{\beta_0}\log(N_0)\left[\log(1-2\chi) - \log(1-\chi)\right] - \frac{3C_F}{\beta_0}\log(1-\chi)\,, \end{aligned}$$

where $\chi = \log(N)\,\alpha_s(m_b)\beta_0/4\pi$, $\Gamma_2 = C_F[C_A(67/36 - \pi^2/12) - 5n_f/18]$, $\beta_0 = (11C_A - 2n_f)/3$, and $\beta_1 = (34C_A^2 - 10C_A n_f - 6C_F n_f)/3$. Γ_2 is the $O(\alpha_s^2)$ piece of the cusp anomalous dimension, which was taken from Ref. [31, 32].

The expression in Eq. (54) gives the resummed expression for the moments of the differential cross section to next-to-leading logarithmic order. To obtain the differential cross section, the inverse-Mellin transform of Eq. (54) must be taken. Using the results of Ref. [33], we find:

$$\begin{aligned} \frac{d\Gamma}{dx} &= -\int_x^1 \frac{d\xi}{\xi}\,\Gamma_0(m_b)\,S(\xi) \qquad (56)\\ &\qquad x\frac{d}{dx}\left\{\theta(\xi - x)\,\frac{\exp[l g_1(\alpha_s\beta_0 l/(4\pi)) + g_2(\alpha_s\beta_0 l/(4\pi))]}{\Gamma[1 - g_1(\alpha_s\beta_0 l/(4\pi)) - \alpha_s\beta_0 l/(4\pi) g_1'(\alpha_s\beta_0 l/(4\pi))]}\right\}, \end{aligned}$$

where $l \approx -\log(\xi - x)$, $\alpha_s \equiv \alpha_s(m_b)$, and the shape function S contains no large logarithms.

6 Conclusion

By introducing soft collinear effective theory within the context of $B \to X_s\gamma$ decay near the endpoint I hope I have been able to shed some light on some of the developments that have taken place in this field recently. There has been much work that I have not been able to cover. In particular I have not discussed the application of SCET to exclusive decays. For an up-to-date discussion of recent progress on exclusive rare and semileptonic B decays I refer you to Dan Pirjol's talk [34].

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